

19/07/2021

Ques Find the root of the equation $x^3 - x - 1 = 0$ lying between 1 and 2 by bisection Method.

Soln

$$f(x) = x^3 - x - 1 = 0$$
$$f(1) = 1 - 1 - 1 = -1 < 0$$
$$f(2) = 8 - 2 - 1 = 5 > 0$$

$\therefore f(1)$ is negative & $f(2)$ is positive there exists at least one real root b/w 1 and 2. Now we apply Bisection Method for first approximation.

$$x_1 = \frac{1+2}{2} = 1.5$$

$$f(1.5) = (1.5)^3 - 1.5 - 1$$
$$= 3.375 - 1.5 - 1 = 0.875 > 0$$

So now the root will lie between 1 and 1.5
So for second approximation.

$$x_2 = \frac{1+1.5}{2} = 1.25$$

$$\therefore f(1.25) = (1.25)^3 - 1.25 - 1$$
$$= 1.953 - 2.25 = -0.297 < 0$$

$\therefore f(1.25)$ is -ve and $f(1.5)$ is +ve the real root will lie b/w 1.25 & 1.5.

$$\therefore x_3 = \frac{1.25+1.5}{2} = 1.375$$

$$\Rightarrow f(1.375) = (1.375)^3 - 1.375 - 1$$
$$= 0.2246 > 0$$

The required root lies b/w 1.25 & 1.375

$$\therefore x_4 = \frac{1.25 + 1.375}{2} = 1.313$$

$$\Rightarrow f(1.313) = (1.313)^3 - 1.313 - 1 \\ = -0.0494 < 0$$

\therefore The required root lies b/w 1.313 & 1.375

$$\Rightarrow x_5 = \frac{1.313 + 1.375}{2} = 1.344$$

$$\Rightarrow f(1.344) = (1.344)^3 - 1.344 - 1 \\ = 0.0837 > 0$$

\therefore The required root lies b/w 1.313 and 1.344.

$$\Rightarrow x_6 = \frac{1.313 + 1.344}{2}$$

$$= 1.329$$

$$\Rightarrow f(1.329) = (1.329)^3 - 1.329 - 1 \\ = 0.0183 > 0$$

\therefore The required root lies b/w 1.313 & 1.329.

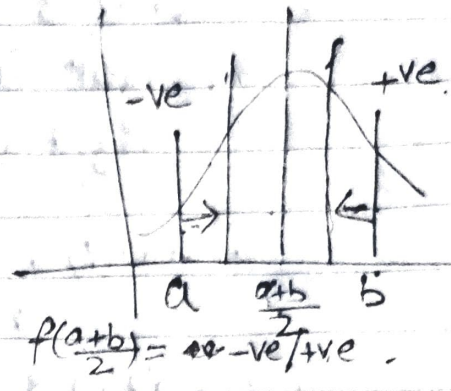
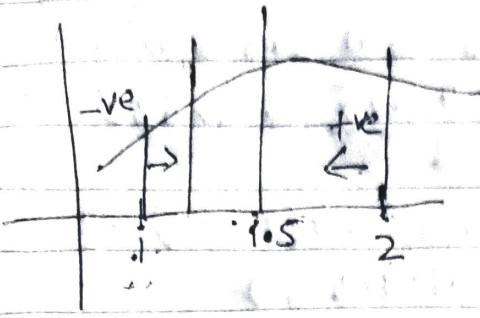
$$\Rightarrow x_7 = \frac{1.313 + 1.329}{2} = 1.321$$

$$\Rightarrow f(1.321) = (1.321)^3 - 1.321 - 1 \\ = -0.0158$$

\therefore The required root lies b/w 1.321 & 1.329.

$$x_8 = \frac{1.321 + 1.329}{2} = 1.325$$

\therefore The root of the given equation is 1.32 for upto two decimal places



Ques Find a root of the equation $f(x) = x^3 - 4x - 9 = 0$ using the bisection Method upto four iterations.

Soln - Given $f(x) = x^3 - 4x - 9 = 0$

$$f(1) = 1 - 4 - 9 = -12 < 0$$

$$f(2) = 2^3 - 4(2) - 9 = -9 < 0$$

$$f(3) = 3^3 - 4(3) - 9 = 6 > 0$$

$\therefore f(2)$ is negative and $f(3)$ is positive the real root lies b/w 2 and 3. Now applying the bisection method we obtain the first approximation.

$$x_1 = \frac{2+3}{2} = 2.5$$

$$f(2.5) = (2.5)^3 - 4(2.5) - 9 = 15.625 - 19 = -3.375 < 0$$

$\therefore f(2.5)$ is -ve & $f(3)$ is +ve the real root lies b/w 2.5 & 3

$$\therefore x_2 = \frac{2.5+3}{2} = 2.75$$

$$\Rightarrow f(2.75) = (2.75)^3 - 4(2.75) - 9 = 20.797 - 20 = 0.797 > 0$$

∴ $f(2.5)$ is -ve & $f(2.75)$ is +ve
the root lies b/w 2.5 & 2.75

$$x_3 = \frac{2.5 + 2.75}{2} = 2.625$$

$$f(2.625) = (2.625)^3 - 4(2.625) - 9 \\ = -1.412 < 0$$

∴ $f(2.625)$ is -ve & $f(2.75)$ is +ve
the root lies b/w 2.625 & 2.75

$$\Rightarrow x_4 = \frac{2.625 + 2.75}{2}$$

$$= 2.6875$$

∴ The real root of given equation at
4th iteration is 2.6875

Q

$$f(x) = 8x^3 - 2x - 1 = 0$$

Soln

$$f(0) = 0 - 0 - 1 = -1 < 0$$

$$f(1) = 8 - 2 - 1 = 5 > 0$$

∴ $f(0)$ is -ve & $f(1)$ is +ve the real
root lies b/w 0 and 1

1st Approximation

$$x_1 = 0.5$$

$$\Rightarrow f(0.5) = 8(0.5)^3 - 2(0.5) - 1 \\ = -1 < 0$$

2nd Approximation

$$x_2 = \frac{0.5 + 1}{2} = 0.75$$

$$f(0.75) = 8(0.75)^3 - 2(0.75) - 1 \\ = 0.875 > 0$$

3rd Approximation

$$x_3 = \frac{0.5 + 0.75}{2} = 0.625$$

$$f(0.625) = 8(0.625)^3 - 2(0.625) - 1 \\ = -0.296 < 0$$

4th Approximation

$$x_4 = \frac{0.625 + 0.75}{2} = 0.687$$

$$f(0.687) = 8(0.687)^3 - 2(0.687) - 1 \\ = 0.219 > 0$$

5th Approximation

$$x_5 = \frac{0.625 + 0.687}{2} = 0.656$$

$$f(0.656) = 8(0.656)^3 - 2(0.656) - 1 \\ = -0.0535 < 0$$

6th Approximation

$$x_6 = \frac{0.656 + 0.687}{2} = 0.668$$

$$f(0.668) = 8(0.668)^3 - 2(0.668) - 1 \\ = 0.048 > 0$$

7th Approximation

$$x_7 = \frac{0.656 + 0.668}{2} = 0.662$$

For approximate value of real root upto two decimal places for given equation is 0.66.

1. Find a root of the following equations using the Bisection method correct to two places of decimal :

(i) $x^3 - 9x + 1 = 0$

(ii) $x^3 - x - 11 = 0$

(iii) $x^3 - 18 = 0$

(iv) $x^3 + x = 1$

(v) $x^3 - 3x - 5 = 0$

(vi) $x^6 - x^4 - x^3 - 1 = 0$

(vii) $x^3 - 5x + 1 = 0.$

2. Find a real root of $x^3 - x = 1$ lying between 1 and 2 by Bisection method. Compute five iteration.

3. Obtain a root, correct to three decimal places for each of equations using Bisection method :

(i) $x^3 + x^2 + x + 7 = 0$

(ii) $x^3 - 18 = 0$

(iii) $x^3 - 5x + 3 = 0$

(iv) $x^3 - x - 4 = 0.$

4. Find a positive root of the equation $xe^x = 1$ which lies between 0 and 1 by Bisection method.
5. Compute real root of the equation $3x + \sin x - e^x = 0$ in a given interval, by Bisection Method.
6. Find a real root of $e^x = 3x$ by Bisection method.
7. Use Bisection method to find out the positive square root of 30 correct to four decimal places.
8. Apply Bisection method to find a root of the equation $x^4 + 2x^3 - x - 1 = 0$ in the interval $[0, 1]$.
9. Transcendental equation is given as $f(x) = 2^x - x - 3$.

Calculate $f(x)$ for $x = -4, -3, -2, -1, 0, 1, 2, 3, 4$ and compute the intervals between which the roots are lying.

ANSWERS

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|--------------|------------|--------------------|------------|
| 1. (i) 2.94 | (ii) 2.38 | (iii) 2.62 | (iv) 0.75 |
| (v) 2.27 | (vi) 1.40 | (vii) 0.201. | |
| 2. 1.324 | | | |
| 3. (i) 0.552 | (ii) 2.621 | (iii) 0.657, 1.834 | (iv) 1.796 |
| 4. 0.567143 | 5. 0.3604 | 6. 1.5121375 | 7. 5.4771 |
| 8. 0.8667605 | | | |
| 9. | | | |

x	:	-4	-3	-2	-1	0	1	2	3	4
f(x)	:	1.0625	0.0125	-0.75	-1.5	-2	-2	-1	2	9

Root lies in $(-3, -2)$ and $(2, 3)$.